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RESEARCH MEMORANDUM

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Abstract:

The distribution of lead time demand is essential for determining reorder points in inventory systems. Usually, the distribution of lead time demand is approximated directly by fitting mean and standard deviation to a particular theoretical probability function. However, in some cases it may be worthwhile to take the demand per unit time and the lead time explicitly into account, in particular when some specific information about one or two of them is available.

In this paper we investigate a special situation where a supplier, who produces on order in fixed production cycli, provides information about the status of the coming production run at specific moments. The retailer can use this information to get a better insight in the lead time process. The information may not only be used to obtain an accurate approximation of the lead time distribution, but it may also be used in the operational ordering decisions. We present a fixed order (s_t, Q)-strategy with a set of reorder points s_t, depending on the virtual lead time t, which follows from the information of the supplier. A Markov model, which analyses a given (s_t, Q)-strategy, is used to quantify the value of the information provided by the supplier. Some numerical examples show that the approach may lead to considerable cost savings compared to the traditional approach where only one single reorder point, based on a 2-moment approximation, is used.

The results of this paper can be used to balance the pros and cons of more frequent exchange of information between a retailer and his suppliers.

Subject Area: Inventory Management, Decision Theory, and Production/Operations Management.

1. Introduction

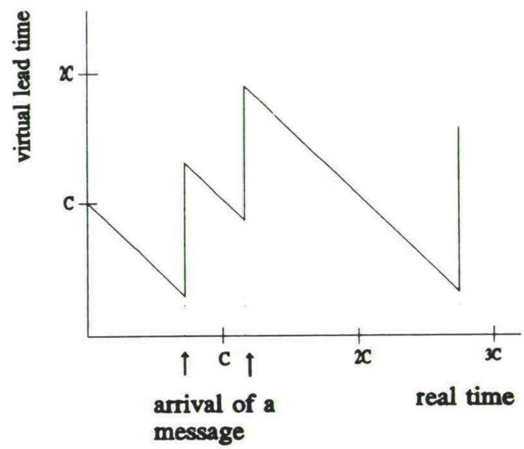
The determination of the distribution of demand during the lead time (or lead time plus review time) is an important issue in the literature on inventory modelling. An excellent survey of literature on this topic is given by Bagchi et al. [1]. Lead time demand can be estimated directly, but can also be estimated by a combination of two factors, namely demand per unit time and lead time, or even by a combination of three factors: order intensity, order size and lead time. Many inventory models emphasize only the variability of demand, and neglect the variability of lead time. In other models, uncertainty in lead times is incorporated by a theoretical distribution which is fitted on the first moments of the lead time (see e.g. Bagchi [2], Carlson [3] and Kottas and Lau [5]) or on the first moments of the demand during lead time (see e.g. Fortuin [4], Silver and Peterson [7] and Tadikamalla [9]). However, as pointed out in several studies (see e.g. Kottas and Lau [6] and Strijbosch and Heuts [8]), a bad approximation of the empirical distribution of the lead time (demand) can have substantial cost consequences. An information system which gathers improved empirical information on the lead time is therefore a very useful management tool.

In many models, the key component of the lead time in which uncertainty exists is the shipping time from the supplier to the stocking point. This reflects the situation where the supplier produces on stock. In this paper, we investigate another practical situation, where the major source of uncertainty, from the retailer's point of view, is the time until the supplier starts a production run to fulfil his order. Here, the supplier produces on order. Nowadays, there is a trend towards more frequent exchange of information between retailers and their suppliers. A practical example of such a cooperation is that the supplier provides information about the status ("full" or "not full") of the coming production run of a particular product. We will show that this information provides a better understanding of the lead time process, which may yield a considerable cost saving.

In particular, we consider a supplier who produces every C (integer) periods a batch of a particular product on order. Deliveries occur only at equidistant points in time 0, C, 2C,... Several retailers place, independent of each other, orders for this product from time to time. An order, placed between time (j-1)C and time jC, is produced in the j'th run if this run is not full. Otherwise the order is delivered at time (j+1)C. The retailers have information about the length of the production cycle (C) and the production time (τ). Moreover, the supplier informs the retailers as soon as a production run is booked up to capacity.

The goal of this paper is to investigate the value of the latter information. The *virtual lead time* t denotes the number of periods before the end of the next production run, *which is not yet full (booked up to capacity)*. So, the virtual lead time decreases in real time by units of one and has upward jumps of size C when a message arrives, stating that the coming production run is filled up or the production has been started (see figure 1).

Figure 1: Illustration of the virtual lead time



Let q_t denote the probability that the next production run will be filled up during the next period given a virtual lead time of t. We assume that $q_t=0$ for $t>C$; so the probability that a production run will fill up before the preceding run has been finished is negligable. The production time of an order equals an integral number of τ periods ($1\leq\tau\leq C$). Order entry for a production run ends as soon as the production starts. The last opportunity to place an order is at time τ ($q_\tau = 1$). Without loss of generality, we use $\tau = 1$ in the rest of this paper. Shipping times from the supplier to the stocking point are neglected.

To assess the information provided by the supplier, we have to know which advantages can be drawn by the retailer from this information. Hence, the problem is to find an appropriate ordering strategy for a retailer which makes use of this information. We restrict attention to fixed order strategies with ordering opportunities occurring at the beginning of every period. We propose the following ordering strategy. For every possible value t of the virtual lead time there exists a reorder point s_t such that a predetermined fixed quantity, Q, is ordered, as soon as the inventory position drops to or below s_t. Note that in traditional models, one single reorder point applies for all values of the virtual lead time. In this special situation, the dependance of the reorder points on the virtual lead times is introduced to take advantage of the information which is given by the supplier.

Demands for the product in subsequent periods are independent identically distributed random variables with a discrete probability distribution $\{ \phi_1(k), 0 \leq k \leq M_1 \}$. The mean and variance of the one-period-demand are given by μ and σ^2, respectively. Excess demands are backlogged.

The retailer wants to minimize the total expected long run average cost per time unit subject to a given service level. We use a service level constraint of P2-type, i.e. at least P2% of demand has to be satisfied directly from stock on hand. Given the structure of the ordering strategy the retailer has still to decide on the values of the reorder points s_t. Since the ordering decision can be delayed until $t=C$ without harm ($q_t = 0$ for $t > C$) we are only interested in s_t for $t = 1,..,C$. The order quantity, Q, is not used as a decision variable. It is predetermined and based on, for example, EOQ-considerations or the capacity of a shipping container. The long run purchasing and ordering cost are not affected by the choice of the decision variables. So the only relevant cost factor is the holding cost.

As far as we know, this problem has not been investigated in the literature. We present a method to quantify the value of information provided by the supplier about the status of the next production run. The performance of a given

(s_t,Q)-strategy is analysed in Section 2 using a Markov-model. The determination of the value of information is presented is Section 3 together with some numerical results. We conclude with some final remarks in Section 4.

2. Analysis of a given (s_t,Q)-strategy

In this section, a given (s_t,Q)-strategy will be analysed as it is seen from the point of view of the retailer. The performance measures of interest are the long run average inventory level (since this determines the inventory holding cost) and the long run fraction of demand which is satisfied directly from stock on hand (the service constraint).

Before starting with the analysis, we claim that in any realistic situation $s_t \geq s_{t-1} \geq 0$ for $t=2,..,C$ (reorder points decrease with decreasing virtual lead time). Moreover, we assume that Q is large enough to avoid the situation that the retailer places two orders in the same production cycle ($Q \geq 2\mu C$).

To analyse the system, we define

X_n:= inventory level of the retailer just after the n'th arrival of a supply.

Then, the embedded stochastic process $\{X_n, n=1,2,..\}$ is a discrete-time Markov chain with a state space $S:=\{ i \mid i=i_0,..,i_N \}$, where i_0 (i_N) denotes the minimal (maximal) physical inventory level under the (s_t,Q)-strategy (e.g. $i_N = s_C + Q$).

For any fixed (s_t,Q)-strategy we define for all states $i \in S$:

T(i):= expected number of periods until the next arrival of a supply, given that the inventory level after the present delivery equals i;

H(i):= expected total number of units on stock until the next arrival of a supply, given that the inventory level after the present delivery equals i;

B(i):= expected number of shortages until the next arrival of an supply, given that the inventory level after the present delivery equals i.

Let p_{ij}, $i,j \in S$, denote the one-step transition probabilities of the Markov chain $\{X_n\}$ and π_i, $i \in S$, its stationary distribution. Finally, define for a given (s_t, Q)-strategy:

$f(s_t,Q)$:= long run average inventory level;
$\alpha(s_t,Q)$:= long run fraction of demand satisfied directly from stock on hand.

From the theory of regenerative processes (see e.g. Tijms [11]), it follows that

$$f(s_t,Q) = \frac{\sum_{i \in S} H(i) \cdot \pi_i}{\sum_{i \in S} T(i) \cdot \pi_i}, \tag{1}$$

and

$$\alpha(s_t,Q) = 1 - \frac{\sum_{i \in S} B(i) \cdot \pi_i}{Q}. \tag{2}$$

Before we derive explicit expressions for T(i), B(i) and H(i), we define

$\phi_t(k)$:= probability that demand during t periods equals k;
$\Phi_t(i,k)$:= probability that demand during t periods equals k and no order is placed during the next t periods, given that the present inventory level equals i and the virual lead time equals t.

$\Phi_t(i,k)$ can be computed recursively for $t=1,..,C$, $i \in S$ and $k=0,..,M_1$ by

$$\Phi_t(i,k) = 1_{\{i>s_t\}} \left\{ \phi_t(k) \left\{ 1_{\{i-k>s_{t-1}\}} + 1_{\{i-k \le s_{t-1}\}} q_t \right\} + 1_{\{i-k \le s_{t-1}\}} (1-q_t) \sum_{j=0}^{k} \phi_1(j) \Phi_{t-1}(i-j,k-j) \right\}, \tag{3}$$

with

$$\Phi_1(i,k) = 1_{\{i>s_1\}} \phi_1(k). \tag{4}$$

To explain formula (3), we consider the case $t=2$. For this special case, formula (3) transforms to

$$\Phi_2(i,k) = 1_{\{i>s_2\}} \left\{ \phi_2(k) \left\{ 1_{\{i-k>s_1\}} + 1_{\{i-k\le s_1\}} q_2 \right\} + 1_{\{i-k\le s_1\}} (1-q_2) \sum_{j=0}^{i-s_1-1} \phi_1(j)\, \phi_1(k-j) \right\} . \tag{5}$$

If $i \le s_2$, then an order will be placed at the beginning of the first period, and $\Phi_2(i,k)$ equals zero, regardless of the demand k. Now, we concentrate on the situation that $i > s_2$. In case $i-k > s_1$, no order will be placed during the next two periods, and $\Phi_2(i,k)$ equals $\phi_2(k)$. If $i-k \le s_1$, then two different scenarios can be distinguished. Firstly, the production run is filled during the first period, which occurs with probability q_2. In this case, no order will be triggered during the next two periods, and $\Phi_2(i,k)$ equals $q_2 \phi_2(k)$. Secondly, consider the case that the production run is not filled during the first period (this occurs with probability $1-q_2$). Then, the inventory level at the beginning of the second period equals i-j with probability $\phi_1(j)$, $0 \le j \le k$. The retailer will not order if $i-j > s_1$. The contribution to $\Phi_2(i,k)$ is then $(1-q_2)\phi_1(j)\phi_1(k-j)$.

$\Phi_t(i,k)$ is a key function in the deriviation of expressions for T(i), H(i), B(i) and p_{ij}. Denote the maximum demand during C periods by M_C. Then, by conditioning on the demand during a production cycle of C periods, it readily follows that

$$T(i) = C + \sum_{k=0}^{M_C} \Phi_C(i,k)\, T(i-k) \ , \tag{6}$$

and

$$B(i) = \begin{cases} \sum_{k=0}^{M_C} \left\{ \phi_C(k)\, [k-i,0]^+ + \Phi_C(i,k)\, B(i-k) \right\} & \text{if } i>0 \ , \\ \sum_{k=0}^{M_C} \phi_C(k)\, k = \mu\, C & \text{if } i \le 0 \ . \end{cases} \tag{7}$$

Note that formula (7) is based on the assumption $s_C \ge 0$.

It is obvious that H(i)=0 for $i \leq 0$. Using the same conditioning argument as above, we obtain

$$H(i) = \sum_{k=0}^{M_C} \{ \phi_C(k)\, V_C(i,k) + \Phi_C(i,k)\, H(i-k) \} \qquad \textit{if } i>0 \,, \tag{8}$$

where

$V_t(i,k)$:= total expected number of items on stock during the next t periods, given that the starting inventory is i, the total demand is k, the virtual lead time is t and no order arrives during these t periods.

$V_t(i,k)$ equals zero if $i \leq 0$. Under the condition that demands occur at the end of the period, $V_t(i,k)$ can be computed for $t>1$ using the following formula

$$V_t(i,k) = i + \sum_{j=0}^{\min(k,i)} \phi_1(j)\, V_{t-1}(i-j,k-j) \qquad \textit{if } i>0 \,. \tag{9}$$

Hence, the quantities $V_t(i,k)$ can be computed recursively starting with $V_1(i,k)=i$ if $i>0$ and $V_1(i,k)=0$ otherwise.

Recall that p_{ij} denotes the probability that the inventory level of the retailer just after the next arrival of an supply equals j, given that the inventory just after the last delivery was i. These one-step transition probabilities are also derived by conditioning on the demand during a production cycle of C periods. Given that demand during one cycle equals k and the starting inventory level is i, an order of Q units will arrive at the end of the current production cycle with probability $\phi_C(k)$-$\Phi_C(i,k)$. The inventory level just after the delivery will equal j if the demand k equals i-j+Q. Given a demand of k units, no order will be placed with probability $\Phi_C(i,k)$, in which case the inventory at the end of the production cycle will equal i-k. Hence, the following recursive relation can be derived:

$$p_{i,j} = \phi_C(i-j+Q) - \Phi_C(i,i-j+Q) + \sum_{k=0}^{M_C} \Phi_C(i,k)\, p_{i-k,j} \,. \tag{10}$$

The stationary distribution can now be obtained by the solution of the set of linear equations $\Pi = \Pi P$ (Π denotes the vector of steady state probabilities π_i and P denotes the matrix of one-step transition probabilities p_{ij}), together with the normalizing equation $\Sigma_i \pi_i = 1$.
It is not possible to derive explicit formulas for the steady state probabilities. However, the set of equations can be solved numerically by standard procedures, such as the iterative method of successive overrelaxation (see Tijms [11]).

Summarizing, for a given (s_t, Q)-strategy, the quantities T(i), B(i) and H(i) are computed recursively from formula (6), (7) and (8). Once the one-step transition probabilities have been obtained by formula (10), the stationary distribution can be found using the simple and fast method of successive overrelaxation. Finally, the long run average inventory level and the long run fraction of demand which is directly met from stock on hand can be computed by formula (1) and (2).

3. Determination of the value of information

In the preceding section a method is presented to compute the performance of a given (s_t, Q)-strategy. This method will be used to quantify the value of information. Recall that the supplier provides information about the status of the next production run of a given product. The retailer receives a message from the supplier as soon as the delivery date of orders, which will be placed in the near future, changes (in this case the virtual lead time increases with C units). Past empirical information can be used by the retailer to estimate the q_t-values, which determine the lead time distribution.

Now suppose that the supplier provides no information about the status of the production cycli. Then, the retailer cannot do better than using a (s_t, Q)-strategy with $s_t = s$ for $t = 1,..,C$ (note that the retailer has no notion about what t or q_t is). Denoting the number of periods before the end of the *current* production cycle by w, $w \in \{1,..,C\}$, the effective lead time of an order which is triggered now equals w if this production run is not yet booked up to capacity, and w+C otherwise.

Define $P_F(w)$ as the probability that the current production cycle is full at the beginning of period w. Since $q_t=0$ for $t>C$, $P_F(C)=0$. The probabilities $P_F(w)$, $w=1,..,C-1$, can be obtained from

$$P_F(w) = 1 - \prod_{t=w+1}^{C} (1-q_t) . \tag{11}$$

Denote the probability that an order is triggered at the beginning of period w by $P_O(w)$. Given the lack of information it seems reasonable to assume that the trigger-moments of orders are uniformly distributed over the production cycle $w=1,..,C$. Hence, $P_O(w)=1/C$ for $w=1,..,C$. Denoting $P_{LT}(j)$ the probability of having a lead time of j periods, then it is easily seen that

$$P_{LT}(j) = \begin{cases} P_O(j)\,(1-P_F(j)) , & \text{for } j=1,..,C, \\ P_O(j-C)\,P_F(j-C) , & \text{for } j=C+1,..,2C-1 . \end{cases} \tag{12}$$

Further, the expected lead time, E_{LT}, and the variance of the lead time, V_{LT}, are given by

$$\begin{aligned} E_{LT} &= \sum_{w=1}^{C} P_O(w) \left\{ w + C\,P_F(w) \right\} , \\ V_{LT} &= \sum_{w=1}^{C} P_O(w) \left\{ P_F(w)\,(w+C)^2 + (1-P_F(w))\,w^2 \right\} - E_{LT}^2 . \end{aligned} \tag{13}$$

Of course, in the situation without information, E_{LT} and V_{LT} cannot be obtained from formula (13), but they are estimated from historical records or subjective estimates by managers. Denote the expectation and variance of demand during the lead time (LT) plus review time (RT) by E_D en V_D respectively, then it's well-known (see e.g. Silver and Peterson [7, p.297]) that

$$E_D = (RT + E_{LT})\,\mu \quad , \quad V_D = (RT + E_{LT})\,\sigma^2 + V_{LT}\,\mu^2 . \tag{14}$$

Note that we use RT=1 in our analysis. As mentioned in the introduction, the distribution of demand during lead time or lead time plus review time is usually approximated by fitting a suitable probability density function, by matching the first two moments of the empirical probability distribution function. The calculation of the reorder point in the inventory system is then based on this theoretical distribution.

Tijms and Groenevelt [10] developed 2-moment approximations of the reorder point in periodic and continuous review (s,S)-inventory systems. They found that normal approximations give very good results with respect to the required service level when $V_D / E_D^2 \leq 0.25$; otherwise good approximations may be found by fitting gamma (or a mixture of Erlang) distributions to the empirical distribution. It is easy to adapt the method of Tijms and Groenevelt, which also takes account of undershoots of the reorder point, for periodic (s,Q)-inventory systems. We will use this method to obtain the reorder point $\hat{s}$, that will be used in case the supplier provides no information. The average inventory level under this $(\hat{s}_t, Q)$-strategy, with $\hat{s}_t = \hat{s}$ for $t=1,..,C$, can be obtained by the method in Section 2.

Denote the fixed order strategy which makes the most effective use of the information of the supplier by (s_t^*, Q). *The value of information*, V_I, is then given by

$$V_I = h\left\{f(\hat{s}_t, Q) - f(s_t^*, Q)\right\}, \tag{15}$$

where h denotes the holding cost per unit per time unit.

To get some insight in the magnitude of the value of information, some numerical examples are presented. We consider the following situations:

- the length of the production cycle equals $C=2$;
- the mean demand per period equals $\mu=4$ with variance σ^2. The standard deviation σ is varied over three levels ($\sigma=1, 2, 4$);
- demands per period follow a mixed Erlang distribution if $\sigma/\mu > 0.5$ and a normal distribution otherwise;
- the production time equals $\tau=1$; so $q_1 = 1$. The value of q_2 is varied over three levels ($q_2 = 0.2, 0.5, 0.8$);
- the predetermined order quantity is varied over two levels ($Q=20, 40$);
- the service level requires that at least 95% of demand is met directly from stock on hand.

The Markov model from the preceding section is used to compare the long run average inventory level of two strategies:

- *strategy S1*: $(\hat{s}_t, Q)$-strategy, with $\hat{s}_t = \hat{s}$ for $t=1,2$. Here, $\hat{s}$ is based on the 2-moment approximation of Tijms and Groenevelt;
- *strategy S2*: (s_t^*, Q)-strategy, where s_t^*, $t=1,2$, is a vector containing the optimal reorder points. The set of optimal reorder points is obtained by an efficient search-procedure, which makes use of the evaluation procedure of Section 2.

All together, we examine 18 examples. The numerical results are presented in Table 1. The percentage (cost) saving on inventory on hand, which can be obtained by an effective use of the information is given by $100 \cdot V_1 / f(\hat{s}_t, Q)$.

Table 1: Numerical examples

			strategy S1			strategy S2			percentage saving on holding cost
σ	Q	q_2	$\hat{s}_1$	$\hat{s}_2$	$f(\hat{s}_t,Q)$	s_1^*	s_2^*	$f(s_t^*,Q)$	
1	20	0.2	13	13	16.66	7	12	12.54	24.73
1	20	0.5	15	15	17.46	7	12	12.52	28.29
1	20	0.8	17	17	18.26	8	12	12.56	31.20
1	40	0.2	9	9	22.66	6	10	20.78	8.29
1	40	0.5	12	12	24.44	6	10	20.67	15.45
1	40	0.8	14	14	25.24	7	10	20.69	18.01
2	20	0.2	14	14	17.14	9	13	13.74	19.86
2	20	0.5	16	16	17.90	10	13	13.78	23.05
2	20	0.8	19	19	19.63	0	14	14.11	28.11
2	40	0.2	10	10	22.90	6	11	21.14	7.69
2	40	0.5	13	13	24.61	7	11	21.18	13.94
2	40	0.8	15	15	25.37	9	11	21.21	16.40
4	20	0.2	20	20	21.55	16	18	18.71	13.16
4	20	0.5	23	23	23.12	14	20	19.20	16.95
4	20	0.8	25	25	23.86	0	21	19.69	17.47
4	40	0.2	15	15	26.24	12	13	23.86	9.08
4	40	0.5	17	17	26.96	12	15	24.27	9.97
4	40	0.8	19	19	27.67	9	16	24.45	11.64

It turns out that the percentage cost saving can be very large (up to 30%). This result corresponds with other studies which pointed out that a misspecification of the distribution of the (demand during the) lead time can have very bad consequences.

Table 1 shows that the percentage cost saving increases if q_2 increases, while the other factors are kept the same. This can be explained by the fact that E_{LT}, V_{LT} and the standard deviation of the lead time increase if q_2 increases. Note that, in our examples, the average inventory level $f(s_t^*, Q)$ does not change substantially when varying the value of q_2. This would imply that the system is rather insensitive to inaccurate estimations of q_2.

With increasing σ, the variability in demand gets more important in comparison with the variability in the lead time. Hence, it is not surprising to see that the percentage cost saving decreases if the coefficient of variation of demand increases.

Table 1 shows also that the size of the order quantity has a large impact on the percentage saving. A larger order quantity leads to less orders per unit time, such that the effect from a better lead time information will be significantly smaller.

We conclude with some remarks on the composition of the value of information. Two effects can be distinguished:

- E1: effect of using the information to obtain an accurate approximation of the lead time distribution;
- E2: effect of using different reorder points for different values of the virtual lead time.

Effect E1

The messages of the supplier provide the data for the estimation of the q_t-values. Once the q_t-values have been estimated, the empirical lead time distribution can be approximated very accurately (see formula (12)). Instead of approximating demand during lead time (plus review time) directly (as Tijms and Groenevelt do), lead time demand can also be decomposed into two components: demand per time unit and lead time. Using the more detailed information about the lead time, instead of just the first two moments of it, may decrease the reorder point $\hat{s}$ while the required service level is still satisfied.

Now, consider the following strategy:

- *strategy S3*: $(\bar{s}_t, Q)$-strategy, with $\bar{s}_t = \bar{s}$ for all t, where $\bar{s}$ is the smallest reorder point for which the required service level is achieved.

Note that the difference between strategies S3 and S1 is that under S3 the information about the values of q_t is explicitly used. Starting with $\bar{s}=\hat{s}$ (this is strategy S1), the Markov model from Section 2 can be used to evaluate with how many units $\bar{s}$ can be decreased without violating the service level constraint.

Table 2 lists the value of $\bar{s}_t$, $t=1,2$, and $f(\bar{s}_t,Q)$ for the 18 examples. The difference $h\cdot\{f(\hat{s}_t,Q)-f(\bar{s}_t,Q)\}$ gives the value of an accurate approximation of the probability distribution function of the lead time.

Effect E2

The information of the supplier is also used to differentiate between situations where the present production cycle is already booked up to capacity or not. In the former case the lead time is C periods more than in the latter case. So, in the daily operations, different reorder points are used for different values of the virtual lead time. The difference $h\cdot\{f(\bar{s}_t,Q)-f(s_t^*,Q)\}$ gives the additional value of using the information of the supplier on an operational level.

Table 2: Decomposition of the percentage cost saving in two effects (E1 and E2)

			strategy S3			percentage saving on holding cost		
σ	Q	q_2	$\bar{s}_1$	$\bar{s}_2$	$f(\bar{s}_t,Q)$	total	E1	E2
1	20	0.2	10	10	13.68	24.73	17.88	6.85
1	20	0.5	11	11	13.49	28.29	22.76	5.52
1	20	0.8	12	12	13.28	31.20	27.26	3.95
1	40	0.2	8	8	21.67	8.29	4.36	3.94
1	40	0.5	9	9	21.48	15.45	12.13	3.32
1	40	0.8	10	10	21.27	18.01	15.71	2.30
2	20	0.2	11	11	14.29	19.86	16.63	3.23
2	20	0.5	13	13	15.05	23.05	15.93	7.12
2	20	0.8	14	14	14.86	28.11	24.27	3.84
2	40	0.2	9	9	21.95	7.69	4.13	3.56
2	40	0.5	10	10	21.77	13.94	11.54	2.40
2	40	0.8	11	11	21.58	16.40	14.95	1.45
4	20	0.2	17	17	18.91	13.16	12.27	0.89
4	20	0.5	19	19	19.60	16.95	15.25	1.70
4	20	0.8	21	21	20.34	17.47	14.78	2.69
4	40	0.2	13	13	24.48	9.08	6.71	2.37
4	40	0.5	15	15	25.20	9.97	6.53	3.44
4	40	0.8	16	16	25.03	11.64	9.54	2.10

In Table 2, the percentage holding cost saving of using strategy S2 instead of S1 is decomposed in the effects E1 and E2. It appears that, as expected, the effect E1 has the largest impact.

4. Concluding remarks

In this paper, we have developed a method for computing the value of information, provided by a supplier, in a fixed order inventory system. The information of the supplier, who produces on order in fixed production cycli, provides a better understanding of the lead time process. The special structure of the lead time process is taken into account explicitly by means of a Markov model, which determines the performance of a given ordering strategy.

Some numerical results show that, for the special situation considered, large cost savings can be achieved if the Markov model is used in the optimization instead of the commonly used 2-moment approximation of the demand during lead time and review time. The savings obtained from a better approximation of the lead time distribution are in general much higher than the savings obtained from an operational use of this information in the ordering process.

On an average, 35 CPU-seconds were needed on a VAX-8700-computer to obtain the optimal (s_t^*,Q)-strategy in our examples. This computer time will centaintly increase if the value of C increases. Note, however, that we can restrict ourselves to strategies of type S3 for larger values of C, since the largest part of the value of information is due to a better approximation of the lead time distribution. Furthermore, in general, the retailer will use this method only once to determine the expected savings of more frequent exchange of information with his supplier.

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